Addendum To: On fibre space structures of a projective irreducible symplectic manifold

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In this note, we prove that every fibre space of a projective irreducible manifold is a lagrangian fibration.

THEOREM **1** *Let X be a projective irreducible symplectic manifold and $f : X \to B$ a fibre space with projective base B. Then f is a lagrangian fibration, that is a general fibre of f is a lagrangian submanifold.*

REMARK. Beauville proves that a Lagrangian fibration is a complete integrable system in [2, Proposition 1]. Thus, a general fibre of a fibre space of a projective irreducible symplectic manifold is an abelian variety.

REMARK. Markshevich states in [4, Remark 3.2] that there exists an irreducible symplectic manifold which has a family of non lagrangian tori. But this family does not form fibration.

PROOF OF THEOREM. Let ω be a nondegenerate two form on X and $\bar{\omega}$ a conjugate. Assume that $\dim X = 2n$. Then $\dim F = n$ [5, Theorem 2], where F is a general fibre of f. In order to prove that F is a Lagrangian submanifold, it is enough to show

$$\int_F \omega \wedge \bar{\omega} A^{n-2} = 0$$

where A is an ample divisor on X. Let H' be an ample divisor on B and $H := f^* H'$. Then

$$\int_F \omega \wedge \bar{\omega} A^{n-2} = c(\omega\bar{\omega} A^{n-2} H^n),$$

where c is a nonzero constant. We shall show $\omega\bar{\omega}A^{n-2}H^n = 0$. By [3, Theorem 4.7], there exists a bilinear form q_X on $H^2(X, \mathbb{C})$ which has the following properties:

$$a_0 q_X(D, D)^n = D^{2n} \quad D \in H^2(X, \mathbb{C}).$$

We consider the following equation,

$$a_0 q_X(\omega+\bar{\omega}+sA+tH, \omega+\bar{\omega}+sA+tH)^n = (\omega+\bar{\omega}+sA+tH)^{2n}.$$

Calculating the left hand side, we obtain

$$a_0(q_X(\omega+\bar{\omega})+s^2 q_X(A)+2sq_X(\omega+\bar{\omega},A)+2tq_X(\omega+\bar{\omega},H)+2stq_X(A,H))^n.$$

Since $\omega \in H^0(X,\Omega_X^2)$ and $A, H \in H^1(X,\Omega_X^1)$,

$$q_X(\omega+\bar{\omega},A) = q_X(\omega+\bar{\omega},H) = 0$$

by [1, Théorème 5]. Thus we can conclude that $\omega\bar{\omega}A^{n-2}H^n = 0$ by comparing $s^{n-2}t^n$ term of both hands sides. Q.E.D.

References

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